

Exemple

$$A = \begin{pmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 2 & 0 \\ -\sqrt{3}/2 & 0 & 1/2 \end{pmatrix}$$

$$P_A(\lambda) = \begin{vmatrix} 1/2 - \lambda & 0 & \sqrt{3}/2 \\ 0 & 2 - \lambda & 0 \\ -\sqrt{3}/2 & 0 & 1/2 - \lambda \end{vmatrix} = (2 - \lambda) \begin{vmatrix} 1/2 - \lambda & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 - \lambda \end{vmatrix}$$

$$= (2 - \lambda) \left((1/2 - \lambda)^2 + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \right)$$

$$= (2 - \lambda) \left(1/4 - \lambda + \lambda^2 + \frac{3}{4} \right) = (2 - \lambda) (\lambda^2 - \lambda + 1)$$

$$\lambda_{1,2} = \frac{+1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

$$\lambda \in \left\{ 2, \frac{1 \pm i\sqrt{3}}{2} \right\}$$

$$E_2 = \ker \begin{pmatrix} -3/2 & 0 & \sqrt{3}/2 \\ 0 & 0 & 0 \\ -\sqrt{3}/2 & 0 & -3/2 \end{pmatrix}$$

$$\begin{pmatrix} -3/2 & 0 & \sqrt{3}/2 \\ 0 & 0 & 0 \\ -\sqrt{3}/2 & 0 & -3/2 \end{pmatrix} \sim \begin{pmatrix} -3 & 0 & \sqrt{3} \\ 0 & 0 & 0 \\ -\sqrt{3} & 0 & -3 \end{pmatrix} \sim \begin{pmatrix} -3 & 0 & \sqrt{3} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} x_3 = 0 \\ x_1 = 0 \\ x_2 \text{ libre} \end{cases} \Rightarrow E_2 = \text{span} \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

$$E_{\frac{1+i\sqrt{3}}{2}} : \begin{pmatrix} -\frac{i\sqrt{3}}{2} & 0 & \sqrt{3}/2 \\ 0 & \frac{3-i\sqrt{3}}{2} & 0 \\ -\sqrt{3}/2 & 0 & -\frac{i\sqrt{3}}{2} \end{pmatrix} \sim \begin{pmatrix} -i & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & -i \end{pmatrix}$$

$$\sim \begin{pmatrix} -i & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{cases} -ix_1 + x_3 = 0 \Rightarrow x_1 = -ix_3 \\ x_2 = 0 \\ x_3 \text{ libre} \end{cases}$$

$$\Rightarrow E_{\frac{1+i\sqrt{3}}{2}} = \text{span} \left\{ \begin{pmatrix} -i \\ 0 \\ 1 \end{pmatrix} \right\}$$

A est diagonalisable ,

$$D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & \frac{1-i\sqrt{3}}{2} & 0 \\ 0 & 0 & \frac{1+i\sqrt{3}}{2} \end{pmatrix}$$

avec $P = \begin{pmatrix} 0 & i & -i \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ et $D = P^{-1}AP$